SOME APPROXIMATIONS OF SIGNALS AND INFORMATIONS

J. DAS

INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

(*Received February 7, 1964*)

ABSTRACT. Some methods for approximating time-varying signals and their information contents are discussed and the consequent reduction in the channel capacity for their efficient transmission is indicated. The graphical techniques used in Z-transform methods are mechanised electronically to approximate the waveform of a live signal, and sampling in the time-frequency plane, in time-plane or in frequency-plane alone approximates the information in a continuous semi-random process, e.g., in speech. The temporal and structural attributes of a signal may also be used in approximating the signal information. These lead to some new methods of modulation and band-compression, e.g. Slope-quantized PCM, Adaptive filter and Downward coding.

INTRODUCTION

In analysis and synthesis of communication systems, in modulation processes, in control and servo systems and in various other applications, it becomes necessary to deal with complicated signals which may not be expressible in compact analytical forms. Even if some analytical forms are possible, the consequent difficulties in solving various integro-differential equations are formidable and hence successful attempts have been made to systematize the numerical calculations in the form of exponential series, time-series and Z-transform. However, these approximate methods, although convergent in processing, are applicable only to the time-limited signals, i.e., the exciting signal must be completely known for all time. For signals which are somewhat random function of time, such as speech, telemetering signals or any continuous information-bearing process, an approximate representation is possible by using their characteristics in the time-frequency plane. In a different class of applications, such as the reduction of the complexity of a signal, as is used in narrow-band coding schemes and the high-efficiency modulation systems, the redundancy of information in the signal is exploited to approximate it and then to economise on the required channel capacity. Here the temporal and structural characteristics of signals are used to determine the required approximation. Thus in all complicated applications, some sort of approximation is necessary to simplify the mechanism of representation and processing of signals and information. Along with a review of some existing approximation techniques, some new methods are proposed here on the time and bandwidth necessary for the efficient transmission of information. Some schemes for such practical realization are discussed below. The methods in general fall under two classes of approximation, viz. (a) approximation of waveforms, (b) approximation of information. In problems of network representation and modulation, the approximation of waveforms has been widely used, whereas in problems of bandwidth compression, say in Television and Vocoders, the approximation of information has found many applications.

Approximation of Waveforms

So far the use of the approximation of waveforms has been limited mostly to dead (time-limited) signals (except perhaps in Δ-modulation). But it should be possible to use the same or similar techniques for live signals as well. Suppose we are required to design a function generator capable of giving a set of arbitrary repetitive waveforms. Without going into the existing methods of using diodes to make a broken-line approximation of the waveforms, we may first approximate the required $f(t)$ with eqs.(6) or (7), and then, construct a set of delayed impulses whose heights are varied as required by the given $f(t)$. On double or triple integration of these impulse trains, we have the approximated $f(t)$ which may be made repetitive also. The actual mechanisation of the process is shown

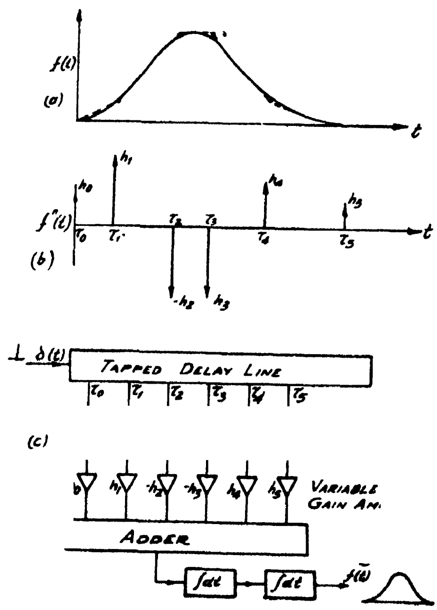

Fig. 3. A function generator and its waveform.
(a) Linear segment approx. of the waveform.
(b) Equivalent second derivative (τ's-time delays and h's -heights of pulses).
(c) Block schematic of the function generator.

in Fig. 3. From the estimation of error in this process and in the use of diodes, we find that this method yields better results with fewer components.

Electronic delay line

It is generally difficult to obtain uniform and distortionless delay for a band of frequencies by using simple LC-networks. On the basis of the staircase approximation of $f(t)$, an electronic delay line, as shown in Fig. 4(a), may be used for many

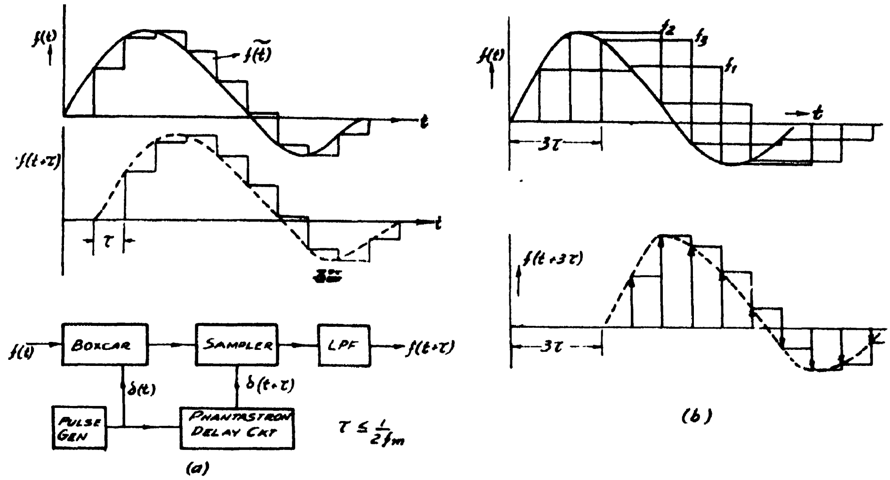

Fig. 4. *An electronic delay line.*

(a) Waveforms and the block schematic for $\tau \leqslant \frac{1}{2f_m}$.

(b) Waveforms for $\tau > \frac{1}{2f_m}$ obtained by sampling in parallel.

applications. Here the staircase wave $\widetilde{f(t)}$ obtained through a Boxcar circuit is sampled again with delayed pulses, the delay τ being less than $\frac{1}{2f_m}$, where f_m is the highest frequency in $f(t)$. Since the sample values remain constant for the time $\tau \leqslant \frac{1}{2f_m}$, the delayed samples, when filtered, will give a delayed $f(t)$ without any distortion. To obtain a delay $n\tau > \frac{1}{2f_m}$, it will be necessary to use cascaded delay circuits, each giving a delay τ less than $\frac{1}{2f_m}$.

Alternatively, $f(t)$ may be sampled in parallel by n Boxcar circuits at time intervals of the required delay $n\tau$, each set of sampling pulses being delayed by $\tau \leqslant \frac{1}{2f_m}$ in a sequence. The resultant staircase waveforms may now be sampled by the delayed sets of pulses, giving n sets of delayed impulses, as seen in Fig. 4(b). The sets are mixed and filtered to give an $f(t)$ delayed by $n\tau$. The technique may also be used for any digitalized signal, e.g., the signal produced by Δ-modulation or PCM.

Orthogonal Filters

An alternative signal approximation, specially for speech, is obtained by using Huggin's (1956) method of signal analysis, as given by eqns.(10-12). Speech signals

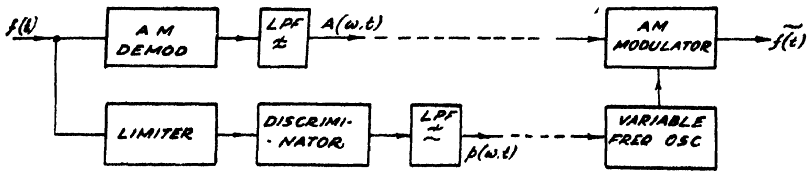

Fig. 11. Approx. of f(t) by AM-FM reduction.

of $f(t)$ are made to modulate the narrow-band exciters in the receivers, the band exciters themselves being controlled by the pitch information obtained from the baseband of $f(t)$. The Vocoders may operate in a very small bandwidth of about 250 c/s under favourable conditions, but it is found that the narrow band filters produce appreciable phase distortions which have audible effects. To eliminate this and to be able to analyze and synthesize entirely in the time domain, cross-correlation Vocoders (David, 1956) have been designed. Here also the correlated amplitudes at fixed time intervals carry the signal information.

To avoid the use of many analyzing bandpass filter, an adaptive filter, shown in Fig. 12, may be used to select a particular formant of $f(t)$. If we assume that

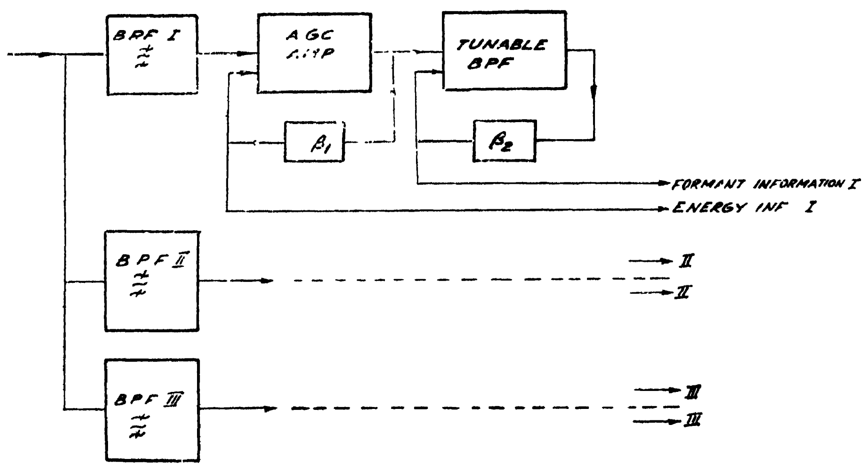

Fig. 12. An adaptive filter for synthesis of f(t).

the signal energy is normally distributed in three or four concentrated bands of the total frequency spectrum (as it happens in speech signals), then each of the band may be automatically selected by an adaptive filter and the control signals, thus derived, may be used to transmit the necessary information to the receiver. For transmission of the six information signals of the process shown, a bandwidth of 300 c/s would generally be required.

Lawrence (1953) has shown that fully intelligible speech would be synthesized out of six slowly-varying parameters, viz. (a) the position of the three principal formants, (b) the intensity of the noise source for consonants, (c) the intensity and frequency of the exponentially decaying source with many harmonics for vowels. The information signals for these synthesized speech require a bandwidth of 150 c/s. only. However the extraction of these basic signals is still a complex problem.

Weighted Modulation

In most of the modulation systems, equal weightage is given to all values of the signal amplitude, and many binary digits are required in PCM to code the complete range of amplitude levels. It is, however, found that the probability of occurence of all signal amplitudes is not same, e.g., the probability of the levels upto the r.m.s. value is 0.5 in speech. Hence there is some scope of increasing the channel capacity (Shannon, 1948) by giving suitable weightage to the different amplitudes corresponding to their probabilities, as is done in the Morse code for English letter. The optimum codes may be produced by using a translator after the Binary or Ternary codes and in the receiver, a complementary retranslation has to be used . However, the complete circuitry would be quite complex to be advantageous for simple applications.

One simple way to obtain some of the advantages of the optimum coding is to compress the signal before processing it for transmission. The compressed signal would contain most of the information of the original signal, and it has been shown that in highly compressed speech (Lickklider *et al.*, 1947), the intelligibility is as good as 95% with the addition of a differentiator at the input. Basically the differentiator accentuates the high frequency components which are mostly responsible for the intelligibility. This result confirms the proposition that the processing of the slopes of $f(t)$, as done in SQ-PCM, will give beter approximation than that obtained from the processing of the sampled amplitudes of $f(t)$. It is expected, then, that many of our modulation methods will produce better results if differentiation and compression proceeds the modulator in the transmiter and the complementary equalization is done in the receiver.

A further possibility of reducing the effective bandwidth for transmission of $\widetilde{f(t)}$ lies in the fact that a digitalized signal may be coded downwards (Bell, 1953) for band-compression, of course at the cost of requiring higher S/N ratio in the transmission path. It has been found, with reference to Fig. 6, that the SQ-PCM gives a good quality speech (0-3 Kc/s band) with 20,000$\pm$ pulses, and a 95%

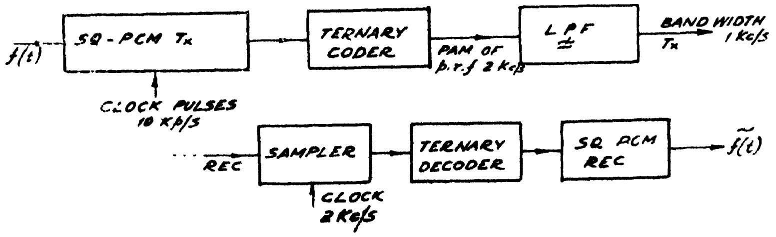

Fig. 13. *Band-Compression by coding downwards.*

intelligible speech may be obtained with as low as 10,000 pulses only. The groups of 5 (or more) pulses may be converted with a ternery code to PAM pulses of much lower p.r.f., as shown in Fig. 13. The filtered wave will then have a bandwidth of 1 Kc/s only for the original 10 Kc/s p.r.f. of the SQ-PCM. In the receiver if the complementary operations are performed, then the message signal would be of much better quality than that obtained by other methods for the same transmission bandwidth. Alternatively the multiplexed Binary PCM or SQ-PCM pulses may be grouped together to form the PAM pulses and transmitted in a reduced video bandwidth. The penalty for this compression by coding downwards would be that a S/N ratio of 40 db in the transmission medium would be required for the 100-level PAM pulses and 60 db for the 1000-level pulses.

DISCUSSION

Most of the schemes shown above aim at reducing the channel capacity or the required bandwidth for efficient transmission of signals. What remains to be decided is an appropriate error criterion so that $\widetilde{f(t)}$ is a good approximation of $f(t)$. This, however, is an open question as the different error criteria, e.g., the r.m.s. error, average error, average differential error etc., are not equally applicable to all situations. If one aims at a good representation of the waveform only, then perhaps the r.m.s. error along with the average error in the derivative of $\widetilde{f(t)}$ will indicate the goodness of the fit. If however, the information content of $f(t)$ is to be approximated, then the cross-correlation between $f(t)$ and $\widetilde{f(t)}$ may give a better indication of the fitness. In speech systhesis, the integrated error in the spectrum along with the subjective intelligibility tests is considered sufficient.

In the existing narow-band coding systems like Vocoders, a compression ratio of 10 : 1 or more has been obtained. It is estimated that for articulate speech, the channel capacity could be reduced from 60,000 bits/sec. to about 2000bits/sec. only. But the necessary technique will surely be very much complicated. In television also, it may be possible to obtain a compression ratio of 20 : 1 as suggested by Gabor (1959), where the compression by "Contour interpolation" would give a factor of 4 to 8 and the equalization of information rate would provide a further factor of 3 to 4. In these systems, the effects of the transmission impairments like frequency and phase distortions, noise and nonlinearities in the transmission path, has not yet been fully determined, But it is expected that with the removal of redundant information in the signal, it will become more susceptive to the external distortion and noise. It is characteristic of the redundant singals, like speech, that they are immune to quite a lot of imperfections in their transmission and instrumentation; and some of these noise-resistant properties have to be sacrificed to gain the reduced transmission bandwidth.

In the methods discussed in earlier sections, only moderate saving in channel capacity is expected. Among the band compression schemes for speech, the sampling in the time domain or in the $t-\omega$ plane would generally give a compression ratio of 3 : 1, while the sampling in the frequency domain may give a ratio of 5 : 1. The adaptive filter would similarly give a compression ratio of 5 : 1 or more, but the technique of coding downwards for a digitalized signal would give ratios higher than 3 : 1 depending upon the perfection obtained in the system instrumentation. In the SQ-PCM proposed here, it has been experimentally determined that for an acceptable speech quality, 20,000 pulses/sec. are sufficient, where as in PCM, a 4-digit code equivalent to a p.r.f. of 30 Kc/s and in Δ-modulation, a p.r.f. of 40 Kc/s would be required. For a better S/N ratio, say, 30db approximately, SQ-PCM requires a p.r.f. of 30 Kc/s, but PCM requires a 5 unit code equivalent to 40 Kc/s and Δ-modulation requires a p.r.f. of 50 Kc/s. Thus in these methods, some amount of compression and gain in channel capacity has no doubt been obtained; and this could be achieved with minimum complexity in instrumentation so that they may have wider application. Even this, if practically realized, would amount to more useful utilization of our existing transmission networks and systems.

ACKNOWLEDGMENT

The author wishes to thank Mr. S. K. Mullick and Mr. M. N. Faruqui for many helpful discussions and for working on some of these schemes. He also records his thanks to Prof. H. Rakshit, D.Sc., F.N.I., for his kind interest in the work.

REFERENCES

Bell, D. A., 1953, "Information Theory", Pitman and Sons.

Das, J., 1961, Electronic Technology, **38**, 298.

David, E. E. Jr., 1956, *Trans. Inst. Radio Engrs.*, *CT*-**3**, 232.

Dudley, H., 1955, *Jour. Audio Engg.*, **3** 170.

Gabor, D., 1959, *del Nuovo Cimento*, **13**, 467.

Goodall, W. M., 1947, *Bell Sys. Tech. Jour.*, **26**, 395.

Guillemin, E. A., 1953, *Proc. Nat. Electronics Conf. America*, **9**, 513.

Huggins, W. H., 1956, *Trans. Inst. Radio Engrs.* *CT*-**3**, 210.

Lawrence, W., 1953, "Communication Theory", Butterworths.

Lerner, R. M., 1959, *Trans. Inst. Radio Engrs.*, *CT*-**6**, 197.

Licklider, J. C. R. and Pollack, I., 1948, *Jour. Acoust. Soc. America*, **20**, 42.

Miller, G. A. and Licklider, J. C. R., 1950, *Jour. Acoust. Soc. America*, **22**, 167.

Schouten, J. F. *et al*, 1952, *Philips Tech. Rev.*, **13**, 237.

Shannon, C. E., 1948, *Bell Sys., Tech. Jour.*, **27**, 397.

Subrahmanyam, D. L. and Peterson, G. E., 1959, *Trans. Inst. Radio Engrs.*, *AU*-**7**, 148.

Truxal, J. G., 1954, *Trans. Inst. Radio Engrs.*, *CT*-**1**, 49.

Tustin, A., 1947, *Jour. Inst. Elec. Engrs.* **94** *pt. II-A*, 130.